American Economic Review: Papers & Proceedings 2008, 98:2, 297–303
http://www.aeaweb.org/articles.php?doi=10.1257/aer.98.2.297

Optimal Life-Cycle Investing with Flexible Labor Supply: A Welfare Analysis of Life-Cycle Funds

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We investigate optimal consumption, asset accumulation, and portfolio decisions in a life-cycle model with flexible labor supply. Using this model, we also investigate the welfare costs of constraining portfolio allocations over the life cycle to mimic popular default investment choices in defined-contribution pension plans.

Most prior work on life-cycle investing has treated labor earnings as exogenous (Viceira 2001; Joao Cocco, Gomes, and Pascal Maenhout 2005; Gomes and Alexander Michaelides 2005; Gomes, Viceira, and Kotlikoff 2006). As such, it has focused on the bond-like feature of labor earnings—the fact that these resources are not closely correlated with the returns to equities—while ignoring the insurance feature of variable labor supply—the ability of investors who do poorly in the market to hedge their losses by working and earning more. Our work considers this second aspect of labor earnings, and studies not only how labor supply affects portfolio choice, but also how portfolio choice affects labor supply. Our framework is a realistically calibrated life-cycle model with wage rate uncertainly, variable labor supply, and portfolio choice over safe bonds and risky equities.

Our analysis reinforces prior findings that equities are the preferred asset for young households, with the optimal share of equities generally declining prior to retirement. However, variable labor supply materially alters preretirement portfolio choice by significantly raising optimal equity holdings. Post retirement, however, the optimal equity share increases as households spend down their financial assets, leaving bond-like pension benefits to increasingly dominate household resources.

Our derived preretirement optimal portfolio allocation is similar to the holdings of "life-cycle" or "target retirement" funds, which are replacing money market and stable funds as the default portfolio in many defined-contribution plans (Zvi Bodie and Jonathan Treussard 2007; Viceira 2008). As we show, it is highly costly for moderately risk-averse investors to invest their savings only in stable value funds. In contrast, the welfare losses from investing in balanced funds (the stock-bond mix is fixed) and life-cycle funds are much smaller and, indeed, negligible in the case of life-cycle funds that follow the average optimal asset allocation path the investor would choose if unconstrained. Interestingly, constraining portfolio choice affects asset accumulation, but has a relatively small effect on labor supply.

Ours is not the first study to incorporate flexible labor supply over the life cycle. Eric French (2005) and Hamish W. Low (2005) explore optimal consumption in a realistically calibrated life-cycle model, but ignore portfolio choice. Bodie, Robert C. Merton, and William F. Samuelson (1992) and Bodie et al. (2004) consider portfolio choice, but assume wages are perfectly spanned by the set of traded securities. Yeung Lewis Chan and Viceira (2000) also consider portfolio choice, but in a less realistic setting.

I. Model

Agents work their first K periods and live a maximum of T periods. Lifespan is uncertain, with p_j denoting the probability of surviving to date j given survival to date $j-1$. Preferences are given by

$$U = E_1 \sum_{t=1}^{T} \delta^{t-1} \left(\prod_{j=0}^{t-2} p_j \right) \frac{(C_t L_t^{\alpha})^{1-\gamma}}{1-\gamma}, \tag{1}$$

where $\delta < 1$ is the discount factor, L_t is time-t leisure, C_t is time-t consumption, $\gamma > 0$ is the

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coefficient of relative risk aversion with respect to consumption, and α is a leisure preference parameter. Leisure is measured as a fraction of total available time and satisfies $L_t \in [\underline{L}, 1]$, where $\underline{L}$ is minimum leisure time (set to 1/3 below). Note that for γ greater than 1—our case of interest—marginal utility of consumption decreases with leisure, thus making leisure and consumption substitutes. With these modified Cobb-Douglas preferences, labor supply is invariant to secular changes in the real wage in accord with US experience.

There are two ways to invest—in riskless bonds with constant gross real return $\bar{R}_f$, and in risky stock, with gross real return R_t. Log stock returns are normally distributed, with mean $\mu + \bar{r}_f$ and variance $\sigma^2{}_R$, where $\bar{r}_f = \ln \bar{R}_f$. Investors hold B_t and S_t dollars of each asset, respectively, and face borrowing and short-sales constraints, so that $B_t \geq 0$ and $S_t \geq 0$. Letting π_t denote the proportion of assets invested in stocks at time t, these constraints imply that $\pi_t \in [0, 1]$ and that wealth is nonnegative. We use R_t^p to denote the after-tax net return on the portfolio held from period t to period $t + 1$, i.e.,

$$(2) \quad R_t^p \equiv 1 + (1 - \tau_C)(\pi_t R_t + (1 - \pi_t)\bar{R}_f - 1),$$

where τ_C is the uniform tax rate applied to all asset income. We ignore tax-exempt retirement accounts, since our focus is on asset allocation, not tax-efficient asset location (see Robert M. Dammon, Chester H. Spatt, and Harold H. Zhang 2004).

The investor starts period t with wealth W_t. He then observes his wage rate w_t and makes work $(N_t = 1 - L_t)$, consumption (C_t), and investment (π_t) decisions. We treat housing and other durables consumption expenditures (h_t) as exogenous, "off-the-top" spending and subtract it from the measure of disposable income.[1] Agents face proportional income taxes. This preserves the scalability/homogeneity of the model and limits the number of state variables. In particular, we assume that labor income is taxed at a rate τ_L, that retirement income is taxed at a rate τ_{SS}, and, as noted, that asset income is taxed at a rate τ_C.

Under these assumptions, the investor's financial wealth at the end of working period t is given by

$$(3) \quad W_{t+1} = R_{t+1}^p(W_t + (1 - h_t)(1 - \tau_L)w_t N_t - C_t),$$

where w_t is the time- t wage.

The log of wages follows the process

$$(4) \qquad \ln w_t = f(t) + v_t + \varepsilon_t,$$

where $f(t)$ is a deterministic function of age, $v_t = v_{t-1} + u_t$, u_t is a permanent shock distributed as $N(0, \sigma_u^2)$, and ε_t is a transitory shock uncorrelated with u_t, which is distributed as $N(0, \sigma_\varepsilon^2)$. The innovation to the permanent component of the wage rate (u_t) can be correlated with the return to equity R_t, with coefficient ρ.

During retirement $(t > K)$, wealth accumulation follows

$$(5) \quad W_{t+1} = R_{t+1}^p (W_t + (1 - h_t)(1 - \tau_{SS})Y - C_t),$$

where Y denotes social security income, which is taxed at a rate τ_{SS}. We assume that the log of social security income is a fraction λ of the average lifetime labor earnings that the agent would have obtained had he worked full time during his working life.

Retirement age and the level of social security benefits are exogenous. In practice, social security income depends on the individual's average earnings in his 35 highest earnings years. French (2003) notes that this provides incentives to retire at age 65 and to increase labor supply over the working life. Thus, our simplified assumption should be viewed as a first-order approximation to the incentives built into the Social Security system.[2]

The agent maximizes (1) with respect to C_t, L_t, and π_t, subject to (2)–(5), $C_t \geq 0$, $L_t \in [\underline{L}, 1]$, and $\pi_t \in [0, 1]$. There are four state variables: age (t), wealth (W_t), and the permanent and transitory components of the wage rate $(\exp(v_t)$, and $\exp(\varepsilon_t))$. However, our assumptions of homothetic preferences and linear tax rates make the model scale free with respect to the permanent component of wages $\exp(v_t)$; i..e, if

[1] Assuming investors save to make a downpayment on a house early in life doesn't materially affect our findngs.

[2] Letting social security income depend on past labor supply decisions—specifically, average past labor supply—introduces a computationally costly extra state variable, but makes little difference to the results.

this state variable doubles, all choice variables double. This allows us to eliminate one state variable by normalizing wealth and the choice variables by $\exp(v_t)$. The model is solved via backward induction using grid search, cubic value function interpolations, and Gaussian quadrature.

II. Calibration: Baseline Results and Comparative Statics

Agents are initially age 21, retire at 65, and die for certain at age 100. Prior to this age we use the mortality tables of the National Center for Health Statistics to parameterize the conditional survival probabilities, p_j for $j = 1, \ldots, T$. We set the discount factor δ to 0.97 and the coefficient of relative risk aversion γ to 5. Following Low (2005), we choose α so that the average labor supply over the life cycle matches the average male hours of work per year reported in the Consumer Expenditure Survey—2,080 hours per annum. Assuming a time endowment of 100 hours per week and that α=0.9, average lifetime labor supply equals 0.374. We take the housing expenditure profile $(\{h_t\}_{t=1}^{T})$ from Gomes and Michaelides (2005).

The mean equity premium (in levels) is set at 4.0 percent per annum, the risk-free rate is set at 1.0 percent per annum, and the annualized standard deviation of innovations to the risky asset is set at 20.5 percent. The tax rate is 30 percent on labor income (τ_L) and 15 percent on retirement income (τ_{SS}). Asset income is taxed at a 20 percent rate (τ_C). These rates roughly match effective income tax rates faced by a typical household.

In order to calibrate the wage income process (4) we combine the wage profile reported in Hans Fehr, Sabine Jokisch, and Kotlikoff (2005), which we use for the deterministic age-dependent component of wages,[3] with the estimates of σ_u and σ_ε of 10.95 percent and 13.89 percent reported in Cocco, Gomes, and Maenhout (2005). The implied wage growth rates over the life cycle generated by this function exhibit an inverted-U shape and are comparable to average total income growth rates in the Panel Study of Income Dynamics (PSID) data. We also assume a zero correlation between stock returns and innovations in the permanent component of wages (ρ). Finally, we set the replacement ratio λ equal to 68.8 percent of labor supply at age 65.

Figures 1, 2, and 3 show baseline results. Figure 1 plots average paths of optimal consumption, income, and financial assets over the life cycle, all relative to permanent income; Figure 2 plots the average path of the optimal allocation to stocks as a percentage of financial wealth; and Figure 3 plots average optimal labor supply before retirement, which occurs at age 65, as a fraction of available hours.

Overall, Figures 1 and 2 show consumption, income, asset accumulation, and asset allocation patterns that are qualitatively similar to those assuming fixed labor supply (Cocco, Gomes, and Maenhout 2005). In particular, consumption, income, and wealth accumulation exhibit an inverted-U shaped pattern over the life cycle, while the share of stocks in the portfolio exhibits a U-shaped pattern.

Figure 3 helps explain the life-cycle pattern of labor income. This figure shows that, consistent with the patterns observed in the data (French 2005; Low 2005), the investor chooses a declining pattern of labor supply over the life cycle after an initial period of slightly increasing labor supply. This pattern, together with the pattern in the wage rate, which in our model, as in the data, exhibits an inverted-U shape, results in income increasing steadily until the investor is in his late thirties, and decreasing smoothly until he reaches retirement age. At that point, income drops by roughly 35 percent, as social security starts replacing labor earnings.

Figure 1 shows that, consistent with the empirical evidence, consumption slightly declines as the investor starts increasing leisure late in his working life, and falls more sharply at retirement, when leisure increases dramatically. Asset accumulation exhibits an inverted U-shape, but assets peak much later than labor income. Assets grow rapidly until the investor is in his mid-fifties, at which point he starts dissaving. The rapid accumulation of assets through middle age reflects concern about wage uncertainty and the presence of liquidity constraints. But portfolio choice also matters here. Figure 2 shows that the investor is optimally fully invested in stocks until his early thirties. At that point, the optimal portfolio share of stocks declines steadily until it reaches

[3] Specifically, we use their earnings function $E(a,2)$, given in equation (9) of their paper, with parameter λ equal to 0. In this function, the argument a denotes age, and 2 denotes the middle-income class.

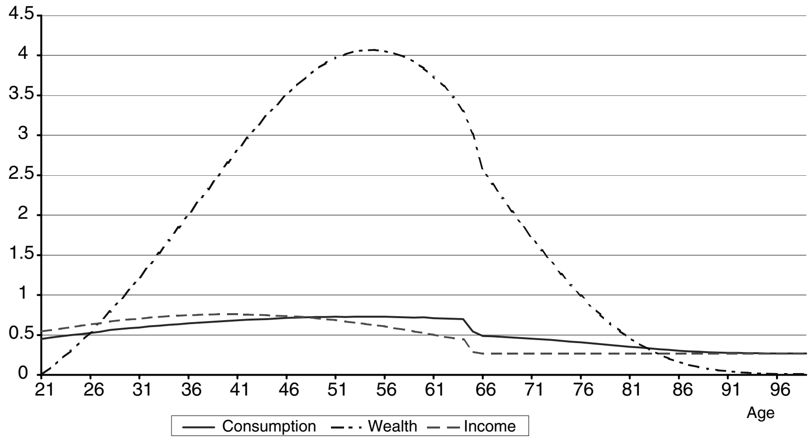

Figure 1. Optimal Consumption, Wealth, and Income

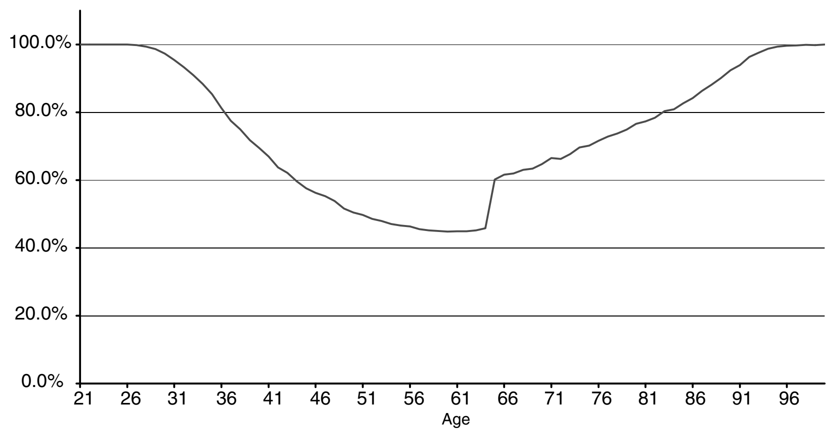

Figure 2. Optimal Portfolio Share Invested in Stocks

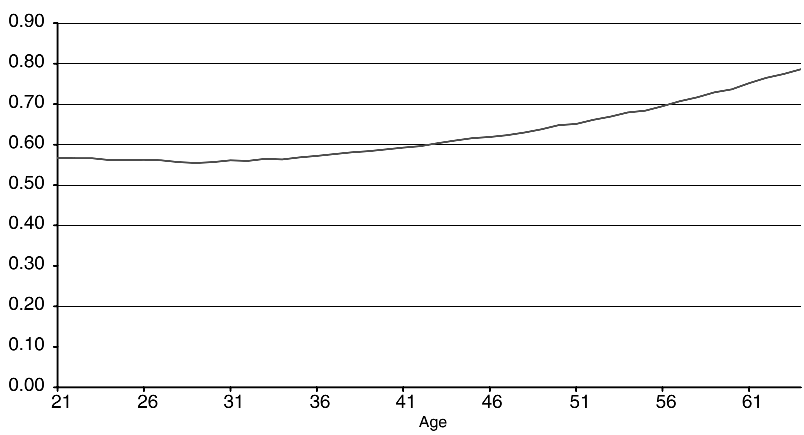

Figure 3. Optmal Leisure

a minimum of about 45 percent at retirement age, and increases monotonically afterward. Thus, while the share of stocks declines steadily during the working life of the investor, it is still very high on average, thus contributing to a rapid growth in asset values along the mean optimal path. The low risk of human capital and its high value relative to financial wealth, both early and late in the life cycle, explain the pattern of the average portfolio allocation to stocks over the life cycle.

The optimal portfolio allocation to stocks over the life cycle generated by our realistically calibrated model is qualitatively similar to the asset allocation path built into self-rebalancing life-cycle mutual funds. Thus, our realistic calibration of life-cycle portfolio decisions and labor supply decisions provides support for this approach to saving for retirement. Our model also suggests that investors receiving pension income should increase their allocation to stocks as they age, as they spend down their assets but experience no diminution of social security income. Note, however, that our model does not account for potentially large financial liabilities generated by health care costs in retirement, which are likely to reduce the investor's willingness to invest in stocks in retirement.

We can now examine the impact on investors' welfare and on optimal decision-making of imposing fixed labor supply constraints and portfolio constraints in our baseline model. Table 1 reports average optimal consumption, wealth accumulation, labor supply, labor income, and portfolio allocation to stocks for each set of constraints (left side of the table), as well as changes in these variables relative to the baseline case (right side of the table). To save space, we report average values of these variables across age ranges. Panel A in Table 1 reports results for our baseline case.

Panel B in Table 1 reports optimal consumption, asset accumulation, and allocation to stocks when labor supply is held fixed to the average labor supply in the unconstrained case. A comparison of panel A with panel B shows that the optimal allocation to stocks is more conservative when labor supply is held fixed. This results from the fact that financial wealth, relative to future labor income, is higher in that case. To understand this pattern, note that panel B shows that labor income is lower early in life than in the case with flexible labor supply, and higher closer to retirement. This is expected, given the roughly declining pattern in optimal labor supply over the life cycle. Interestingly, the individual also chooses a lower level of consumption early in life, which together with higher labor earnings lead to significantly larger wealth accumulation during his working life. This wealth accumulation results in more conservative portfolio allocations over the life cycle, and it sustains higher consumption in retirement.

These results suggest that the ability to increase labor supply acts as an important buffer against future income uncertainty. When we eliminate this extra choice variable, the individual is forced to accumulate extra savings to increase his buffer stock, and behaves more conservatively in his portfolio decisions. The welfare loss from not being able to adjust labor supply optimally is very large. Relative to our baseline model, the investor would be willing to give up 82 percent of his first-year expected labor income to be able to optimally adjust his labor supply. Note that we use first-year labor income as a benchmark for our welfare computations instead of consumption, as is standard in this literature, because we also have leisure entering the utility function. In a model without leisure, the welfare loss in this case would probably correspond to about 4 percent of annual consumption, but in our model we can't make those calculations.

Panel C through panel F examine the impact on consumption, wealth accumulation, and labor supply of constrained portfolio allocations. These allocations mimic investments in a bond (or "stable value") fund (panel C), two balanced funds (panel D and panel E), and a life-cycle fund (panel F), and thus let us explore the welfare costs of popular default choices for defined contribution plans.

Panel C reports results for the case that constrains the investor to invest only in bonds. This is the case considered in prior research on life-cycle consumption with flexible labor supply. Thus, it provides a useful point of comparison for our baseline case. This case is also relevant for its practical relevance, since until recently the preferred default investment choice in defined contribution plans was a money market fund or a stable value fund. Relative to the case where the individual has stocks available for investment, this case leads to significantly lower asset accumulation and consumption over

TABLE 1

	Optimal values					Change relative to baseline case (%)				
Age	C	W	L	Y	π	C	W	L	Y	π
Panel A. Baseline case										
21–30	0.519	0.500	0.561	0.621	0.995					
31–40	0.638	1.929	0.572	0.739	0.828					
41–50	0.707	3.413	0.618	0.735	0.578					
51–65	0.708	3.811	0.733	0.552	0.472					
66–80	0.439	1.562	1.000	0.267	0.678					
81–100	0.291	0.116	1.000	0.266	0.913					
Panel B. Fixed labor supply (Welfare loss = 82.00% of first-year labor income)										
21–30	0.452	0.542		0.565	0.993	−12.9%	8.4%		−9.0%	−0.2%
31–40	0.557	2.167		0.682	0.772	−12.6%	12.3%		−7.7%	−6.8%
41–50	0.667	4.061		0.720	0.521	−5.7%	19.0%		−2.0%	−9.8%
51–65	0.717	5.533		0.619	0.400	1.3%	45.2%		12.2%	−15.2%
66–80	0.573	3.169		0.267	0.530	30.5%	103.0%		0.0%	−21.8%
81–100	0.328	0.296		0.266	0.843	12.7%	155.3%		0.0%	−7.6%
Panel C. Flexible labor supply and 100% bond allocation (Welfare loss = 45.94% of first-year labor income)										
21–30	0.515	0.489	0.558			−0.7%	−2.1%	−0.6%		
31–40	0.624	1.865	0.569			−2.1%	−3.3%	−0.5%		
41–50	0.693	3.249	0.607			−1.9%	−4.8%	−1.7%		
51–65	0.692	3.580	0.719			−2.2%	−6.1%	−1.9%		
66–80	0.419	1.350	1.000			−4.5%	−13.5%	0.0%		
81–100	0.282	0.072	1.000			−3.1%	−37.5%	0.0%		
Panel D. Flexible labor supply and fixed 50/50 stock/bond allocation (Welfare loss = 4.84% of first-year labor income)										
21–30	0.518	0.491	0.560		0.500	−0.1%	−1.8%	−0.3%		
31–40	0.634	1.871	0.570		0.500	−0.5%	−3.0%	−0.3%		
41–50	0.703	3.310	0.616		0.500	−0.5%	−3.0%	−0.3%		
51–65	0.705	3.755	0.730		0.500	−0.4%	−1.5%	−0.4%		
66–80	0.436	1.547	1.000		0.500	−0.6%	−0.9%	0.0%		
81–100	0.289	0.110	1.000		0.500	−0.5%	−5.3%	0.0%		
Panel E. Flexible labor supply and fixed 60/40 stock/bond allocation (Welfare loss = 7.25% of first-year labor income)										
21–30	0.519	0.491	0.560		0.600	−0.0%	−1.8%	−0.2%		
31–40	0.636	1.873	0.571		0.600	−0.3%	−2.9%	−0.2%		
41–50	0.704	3.330	0.618		0.600	−0.4%	−2.4%	0.1%		
51–65	0.708	3.812	0.732		0.600	−0.0%	0.0%	−0.1%		
66–80	0.441	1.607	1.000		0.600	0.4%	2.9%	0.0%		
81–100	0.292	0.125	1.000		0.600	0.4%	8.2%	0.0%		
Panel F. Flexible labor supply and fixed optimal asset allocation (Welfare loss = 2.42% of first-year labor income)										
21–30	0.519	0.500	0.561		0.995	−0.0%	−1.1%	−0.2%		
31–40	0.638	1.929	0.572		0.828	−0.1%	−0.9%	0.1%		
41–50	0.707	3.413	0.618		0.578	−0.1%	−0.6%	0.1%		
51–65	0.708	3.811	0.733		0.472	−0.0%	−0.3%	−0.2%		
66–80	0.439	1.562	1.000		0.678	−0.1%	0.3%	0.0%		
81−100	0.291	0.116	1.000		0.913	0.2%	5.5%	0.0%		

the life cycle, particularly at retirement, and to substantial welfare losses, on the order of 46 percent of first-year labor income.

Panel D and panel E examine the case where investors can hold stocks, but only in fixed proportions of their financial wealth—50 percent and 60 percent, respectively. Balanced funds typically follow this type of fixed-proportion asset allocation strategy with continuous rebalancing. Relative to our baseline case, this constrained case leads to smaller losses in consumption and wealth accumulation than the case with no stock investment at all. Overall welfare losses are also substantially smaller, at 4.8 percent and 7.3 percent of first-year labor income, respectively.

Finally, panel F examines the case where the investor follows a strategy of constantly rebalancing his portfolio using weights that change with age along a deterministic path that equals the optimal average allocation in the unconstrained case (see panel A). For ages below the retirement age, this fixed path mimics the strategy typically followed by life cycle—or target retirement—funds. This strategy is the one that produces minimal deviations in consumption and wealth accumulation with respect to the baseline case, and results in the smallest welfare loss, at 2.4 percent of first-year labor income. We have also computed, but not reported here to save space, the welfare losses for each of these cases when labor supply is fixed. These losses are generally large, but comparable to those with flexible labor supply.

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